

Computational  
Clinical Science Lab

# Mathematical and Computational Modeling of Suicide as a Complex Dynamical System

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Dept. of Psychology, Yale University



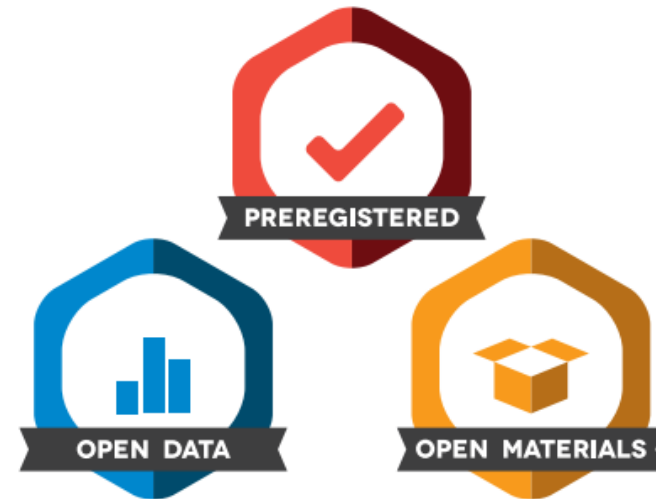
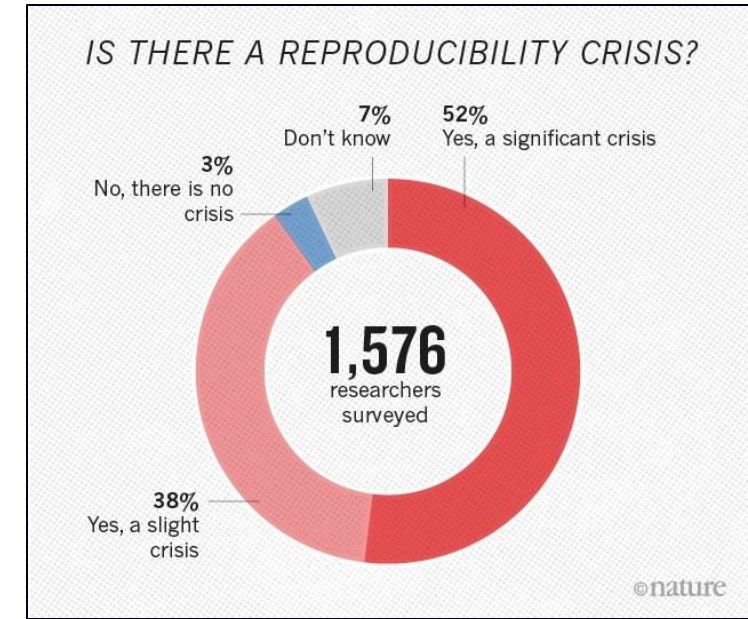
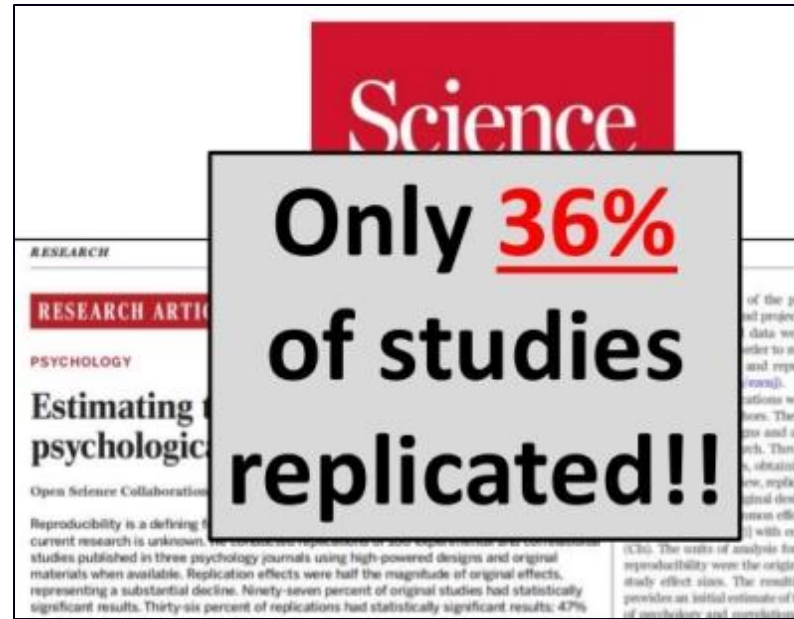
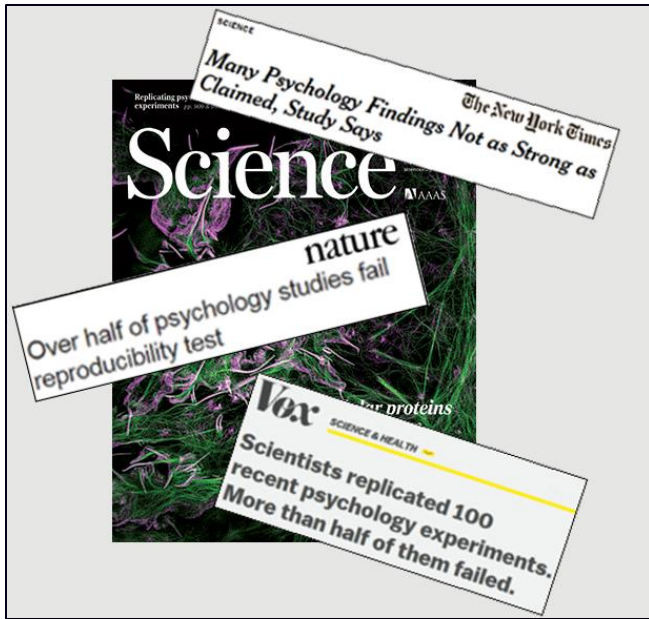
[ccslab.yale.edu](http://ccslab.yale.edu)



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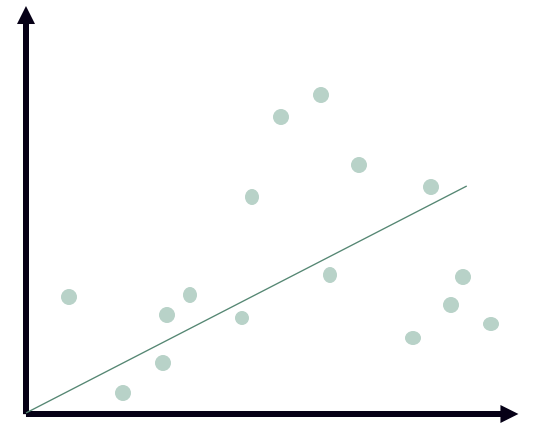


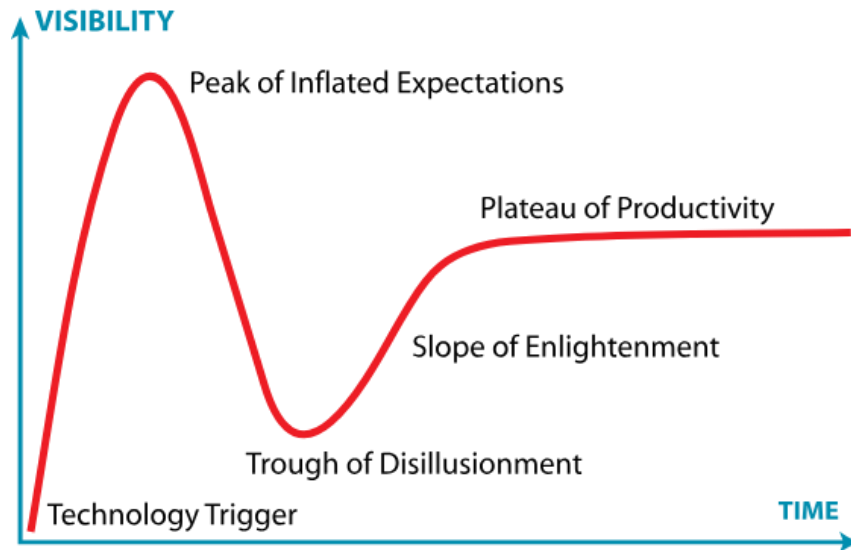
*"Hopelessness leads  
to suicidal  
thoughts."*

**Scientific  
Theories**

**Empirical  
Research**

$p < .05!$





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ISSN: 0033-2909

Psychological Bulletin

2020, Vol. 146, No. 12, 1117–1145  
<http://dx.doi.org/10.1037/bul0000305>

## Interventions for Suicide and Self-Injury: A Meta-Analysis of Randomized Controlled Trials Across Nearly 50 Years of Research

*“prediction was only slightly better than chance for all outcomes; no broad category or subcategory accurately predicted far above chance levels; **predictive ability has not improved across 50 years of research**”*

Psychological Bulletin

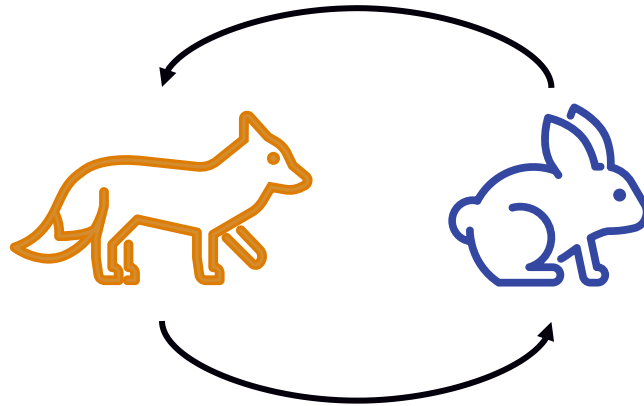
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0033-2909/16/\$12.00 <http://dx.doi.org/10.1037/bul0000084>

## Risk Factors for Suicidal Thoughts and Behaviors: A Meta-Analysis of 50 Years of Research

*“**across five decades, intervention efficacy has not improved**; all interventions produced similarly small effects, and no intervention appeared significantly and consistently stronger than others”*

# Verbal vs. *Formal* Theories

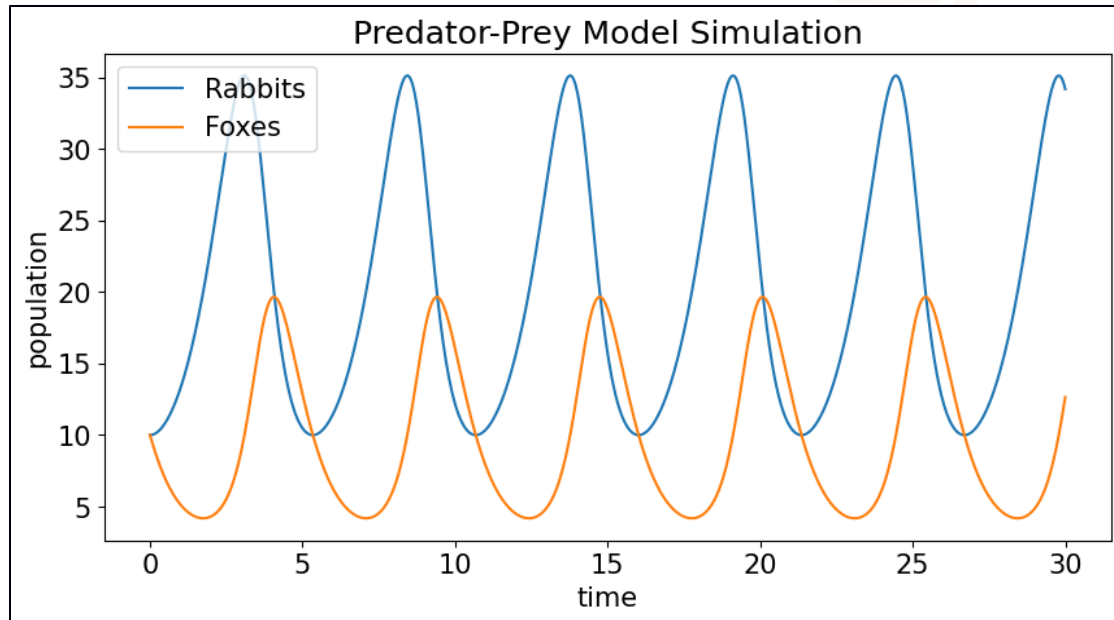
"The rate of change in fox & rabbit populations is related to their current population size and frequency of interaction."



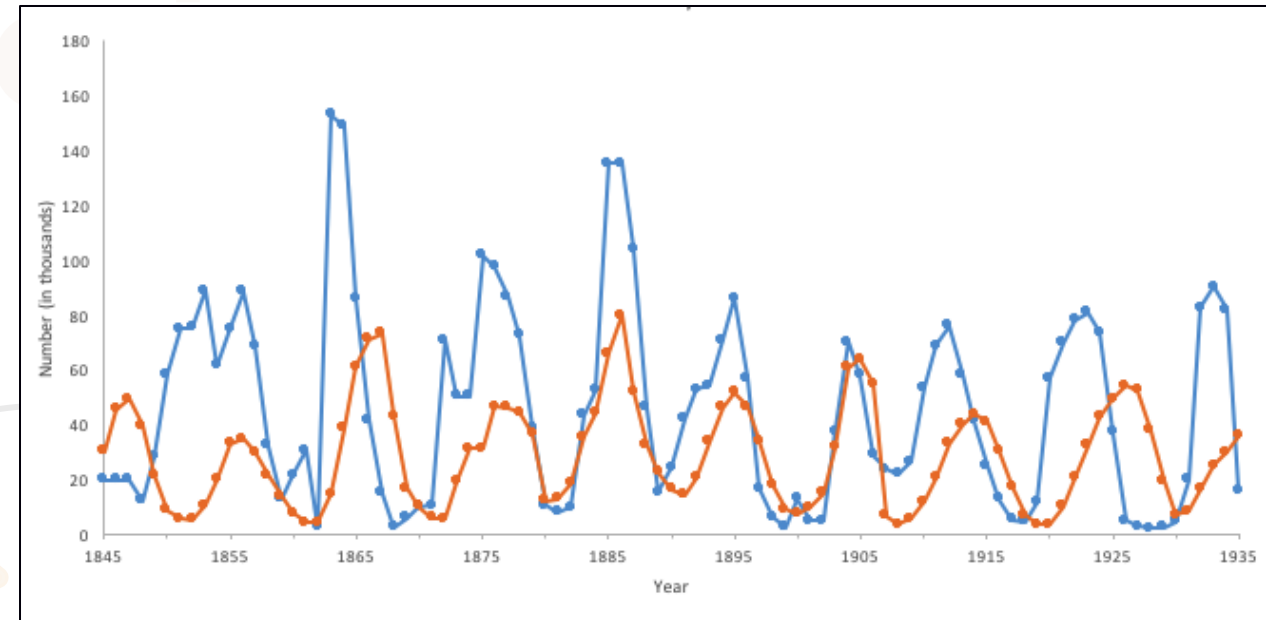
$$\frac{dR}{dt} = \alpha R - \beta RF$$
$$\frac{dF}{dt} = -\gamma F + \delta RF$$

```
def mod(X, t, a, b, c, d):  
    dR = a*X[0] - b*X[0]*X[1]  
    dF = -c*X[1] + d*X[0]*X[1]  
    return(dR, dF)  
  
odeint(mod, X, t, args=(a, b, c, d))
```

# Verbal vs. *Formal*

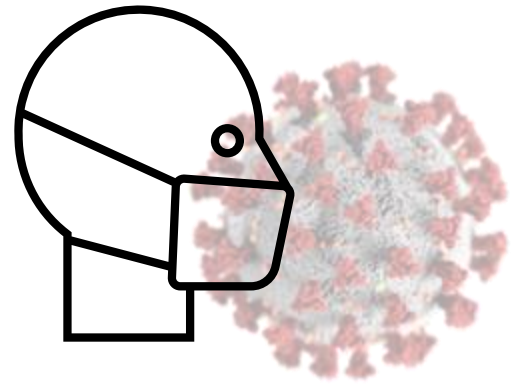
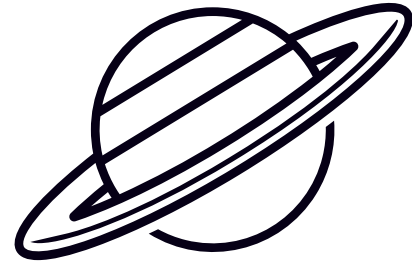
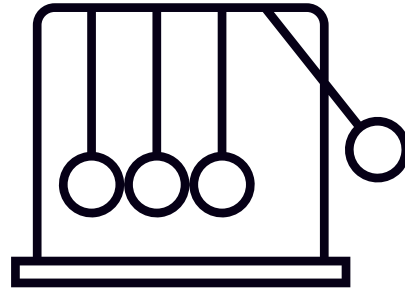
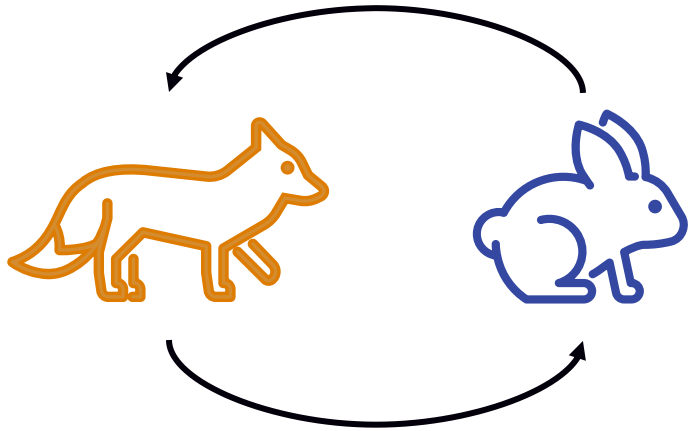


**Theory-Implied Data**



**Empirical Data**

# *Formal theories* guide scientific and clinical discovery



***Formal theories*** guide scientific  
and clinical discovery

**Today:**

Formalizing a Theory of Suicide



# Building a formal theory of suicide

## *Step 1: Starting with External Stressors*

mathematical model  
(formalization)

$$s_t = s_0 \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t\right)$$

verbal theory



# Building a formal theory of suicide

## *Step 1: Starting with External Stressors*

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predictable trends (e.g., chronic,  
systemic, or structural stressors)

verbal theory



# Building a formal theory of suicide

## *Step 1: Starting with External Stressors*

**mathematical model  
(formalization)**

$$s_t = s_0 \exp \left( \left( \left( \mu - \frac{\sigma^2}{2} \right) t + \sigma W_t \right) \right)$$

predictable trends (e.g., chronic, systemic, or structural stressors)

unpredictable events (e.g., acute, episodic, or transient stressors)

**verbal theory**



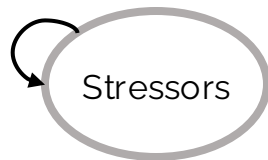
# Building a formal theory of suicide

## *Step 1: Starting with External Stressors*

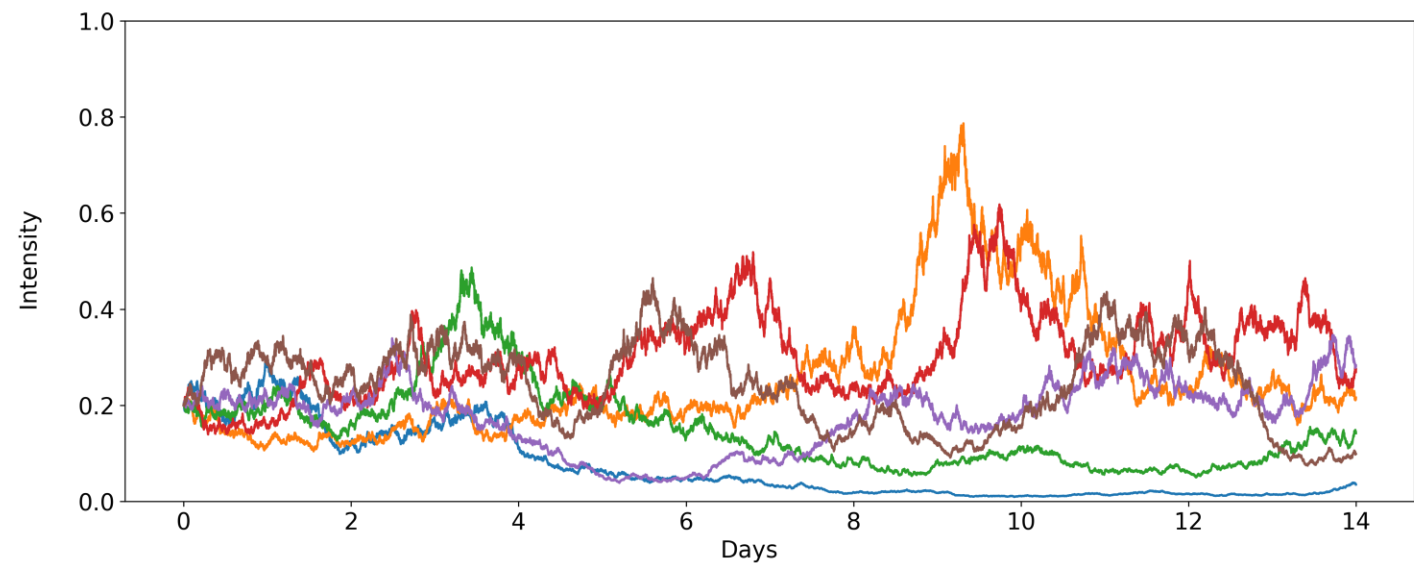
mathematical model  
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verbal theory



theory-implied data  
(simulations/realizations)

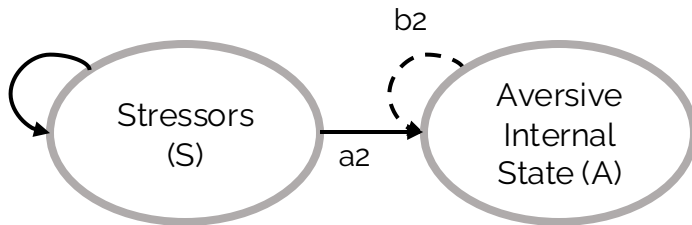


# Building a formal theory of suicide

## *Step 2: Aversive Internal States*

$$S_t = S_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t}$$

$$\frac{dA}{dt} = b_2 A(K_2 - A) + a_2 S$$



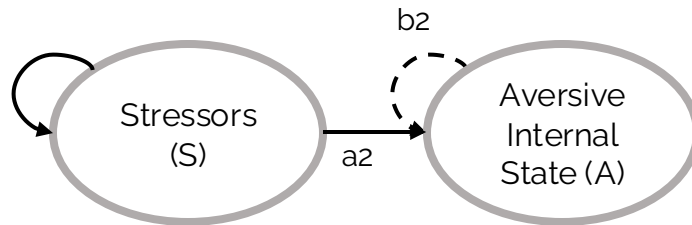
# Building a formal theory of suicide

## *Step 2: Aversive Internal States*

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between-person differences  
(e.g., negative affectivity)

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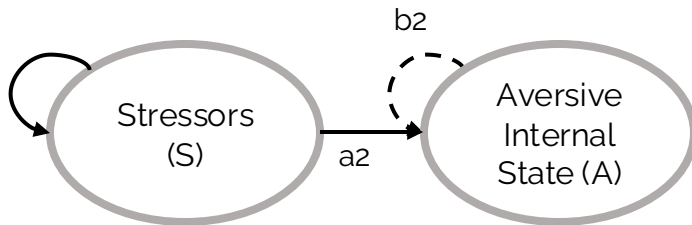
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within-person fluctuations  
(e.g., emotional reactivity)



# Building a formal theory of suicide

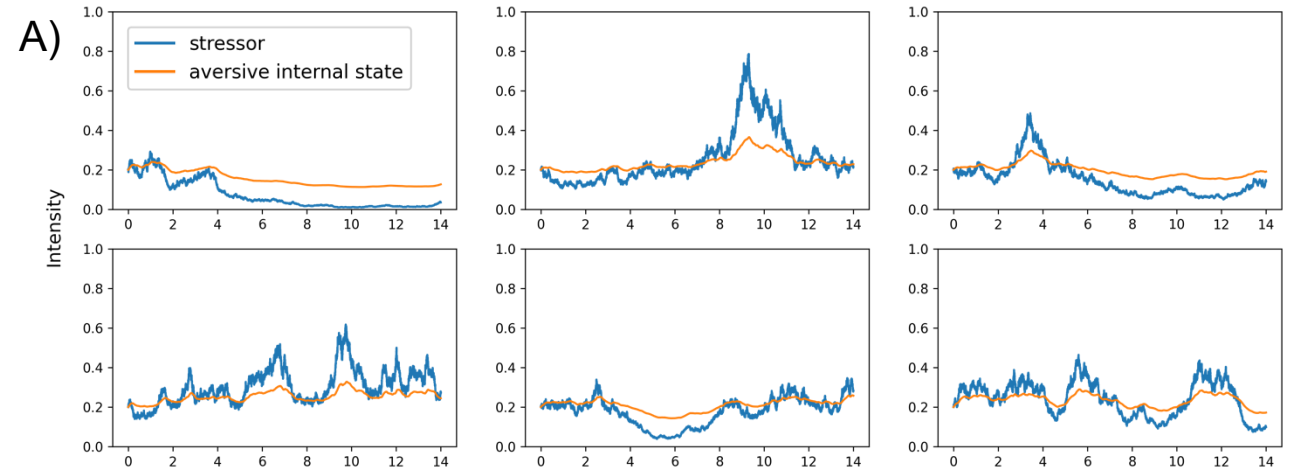
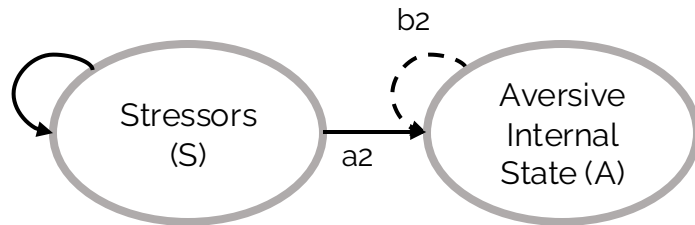
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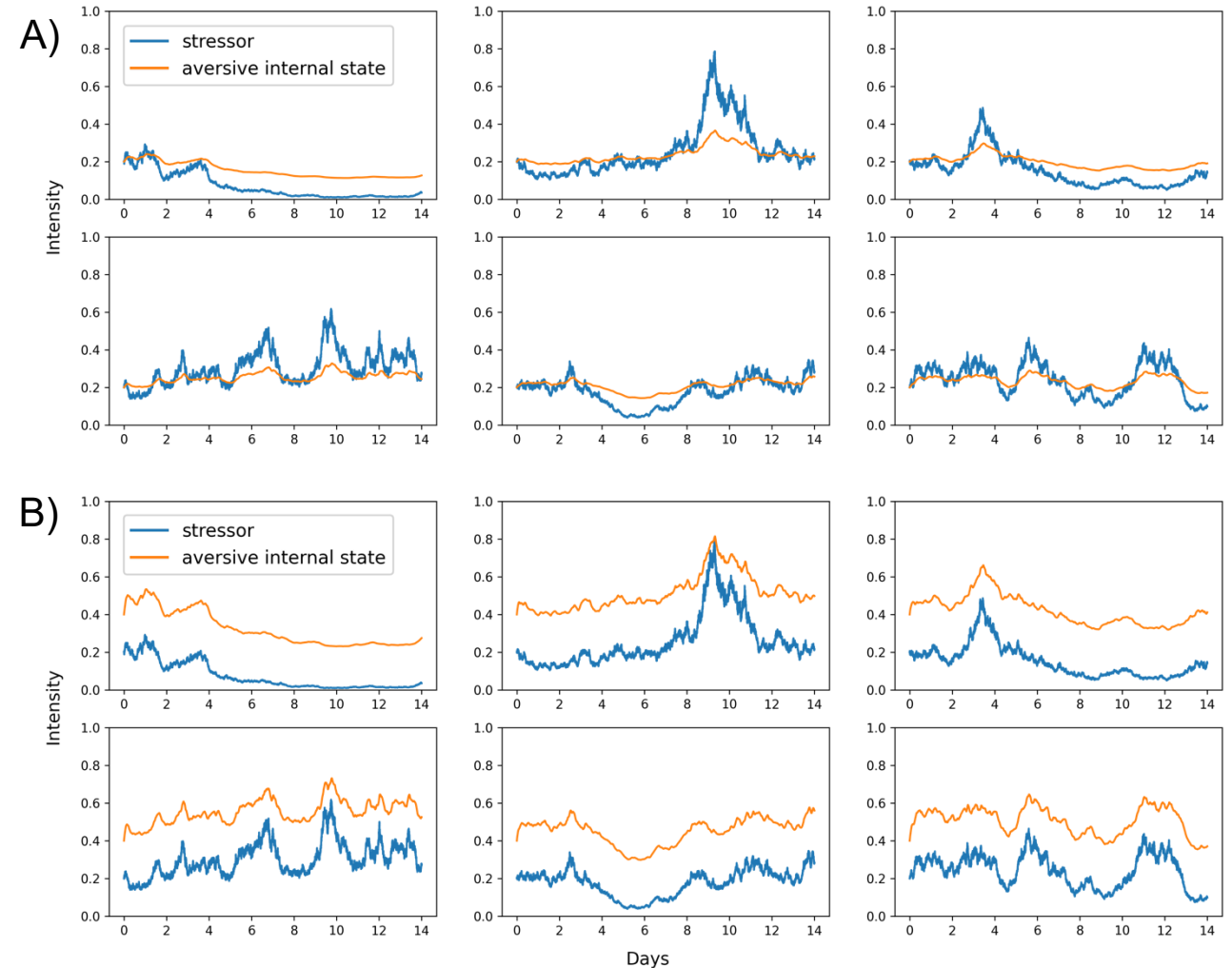
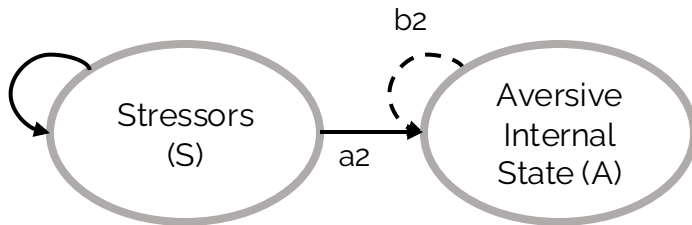
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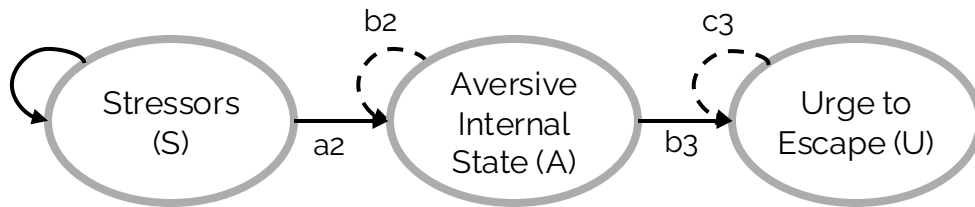
# Building a formal theory of suicide

## *Step 3: Urge to Escape*

$$S_t = S_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t}$$

$$\frac{dA}{dt} = b_2 A (K_2 - A) + a_2 S$$

$$\frac{dU}{dt} = -c_3 U + b_3 A$$



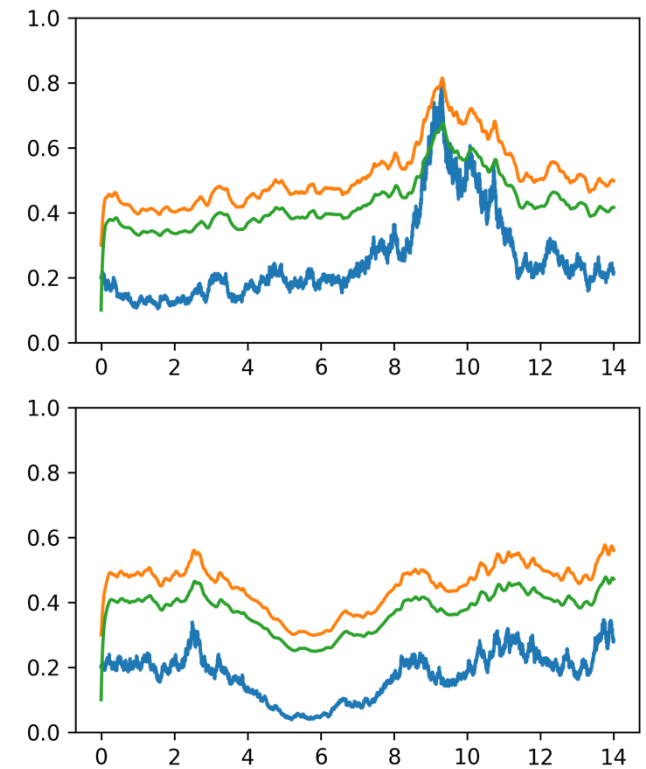
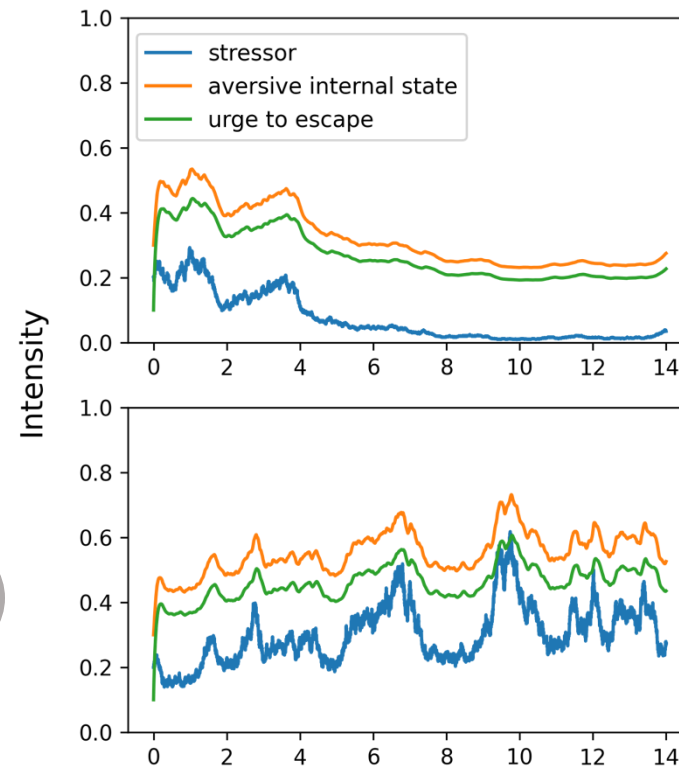
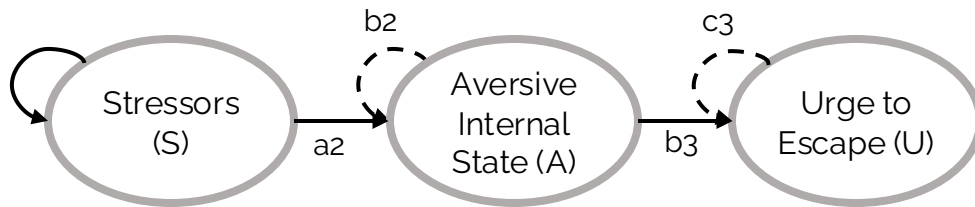
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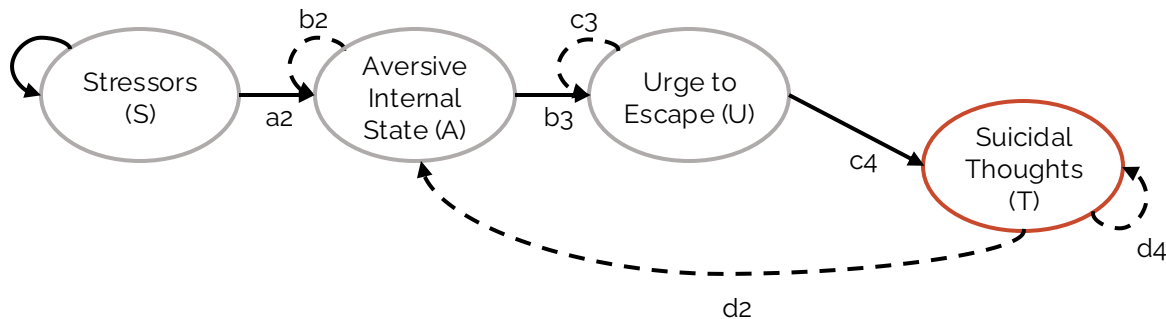
## Step 4: Suicidal Thoughts

$$S_t = S_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t - f_1 E}$$

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# Building a formal theory of suicide

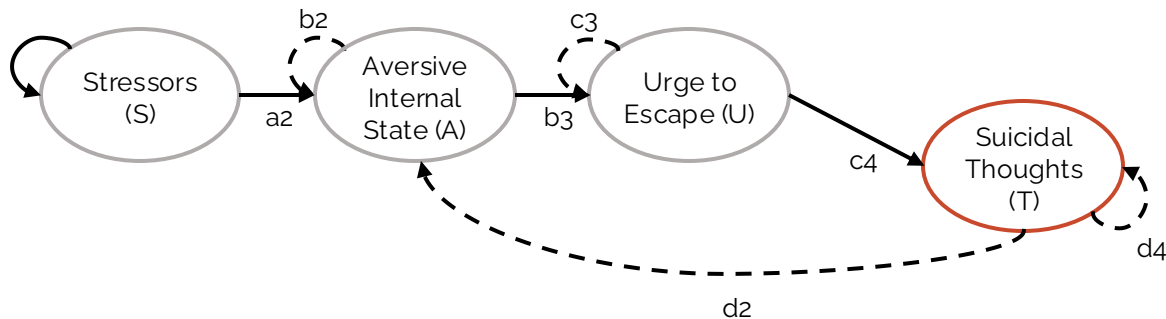
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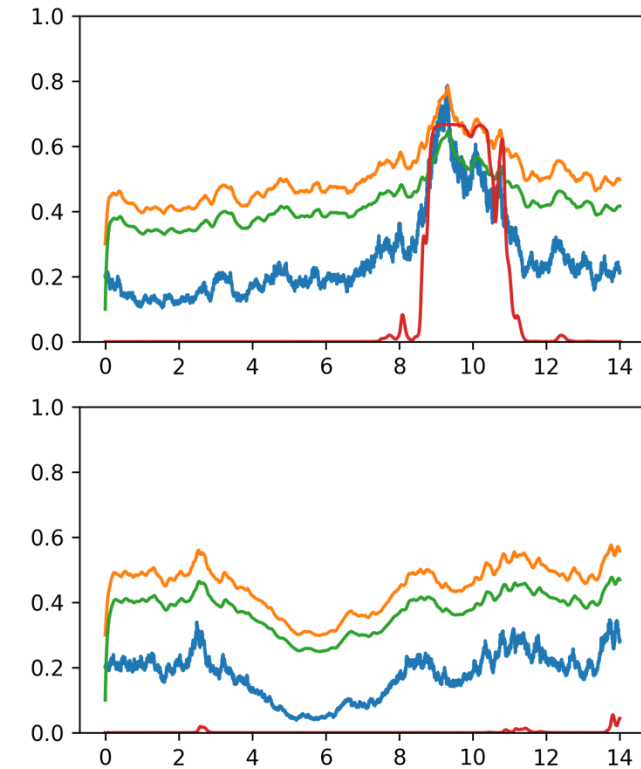
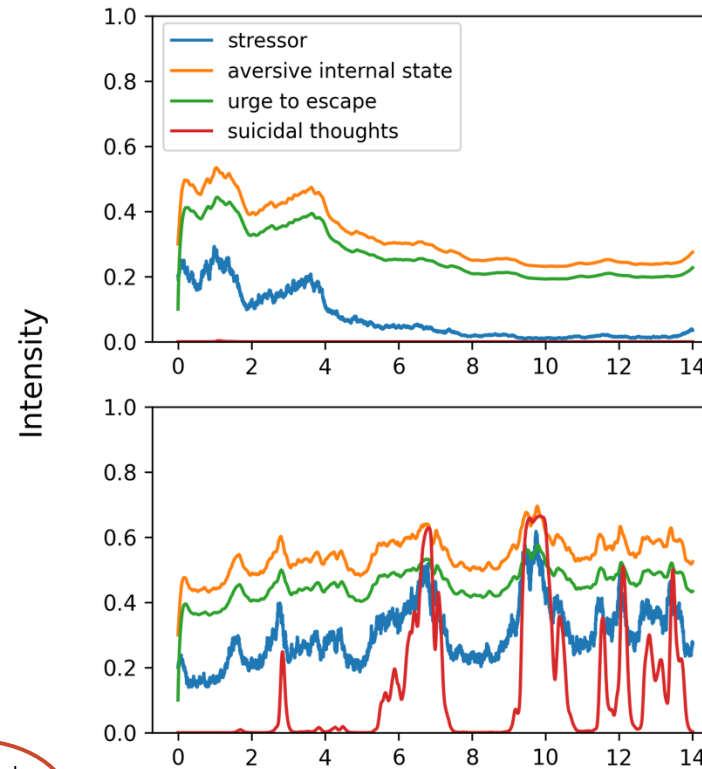
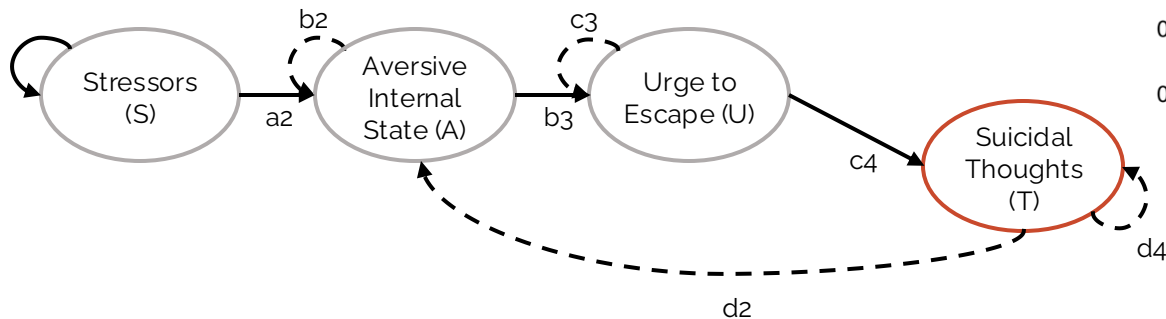
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# Building a formal theory of suicide

## *Final Step (for today): Escape behaviors*

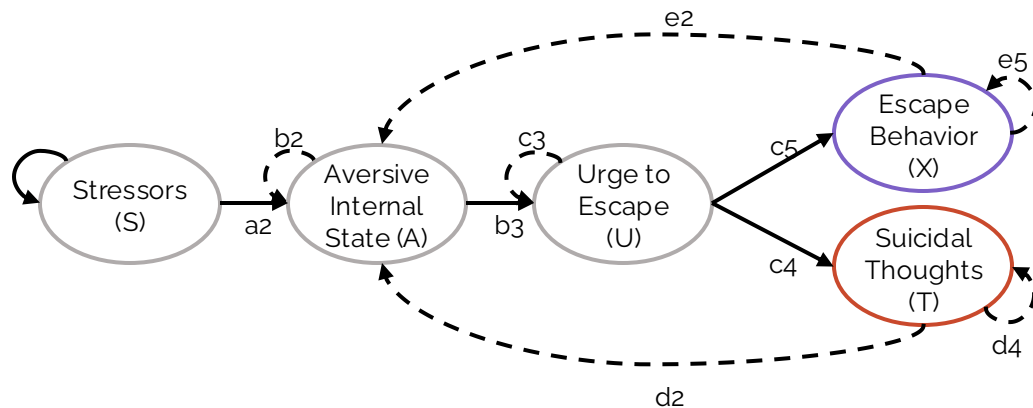
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$$\frac{dX}{dt} = -e_5 X + \frac{1}{1 + e^{-c_{51}(U - c_{52})}}$$



# Simulating a case

*Escape behaviors ARE effective*

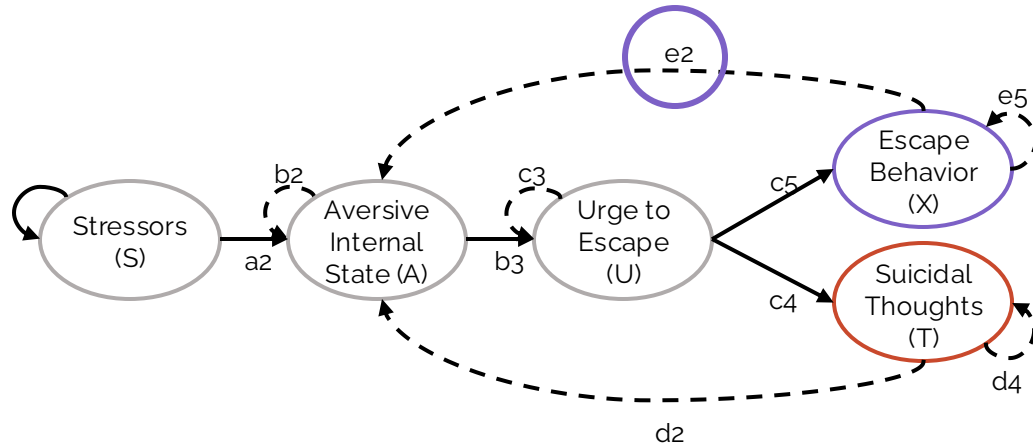
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# Simulating a case

Escape behaviors ARE effective: no suicidal thoughts

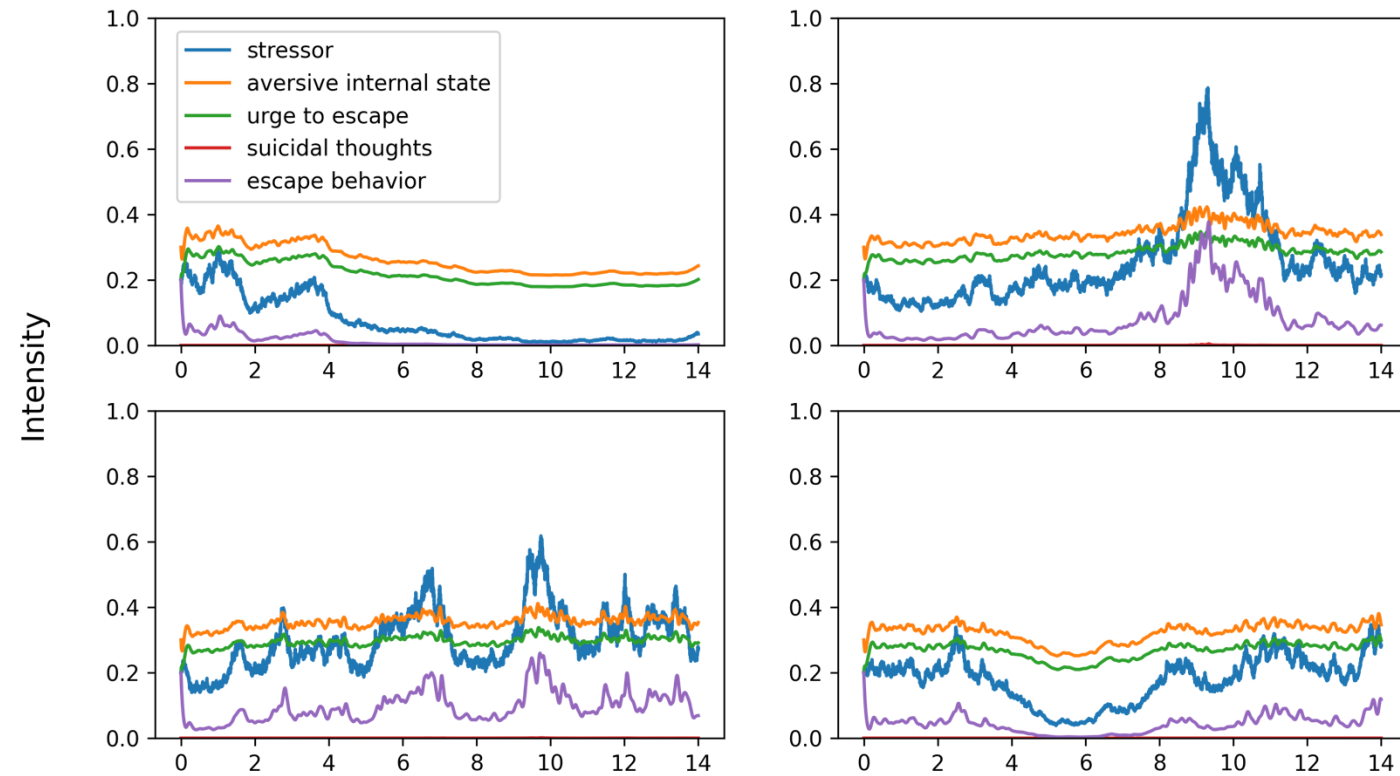
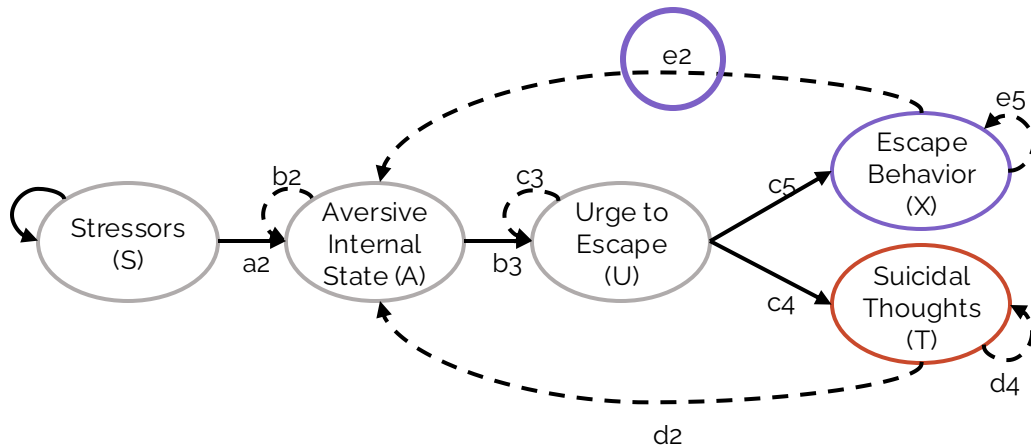
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# Simulating a case

Escape behaviors ARE NOT effective:

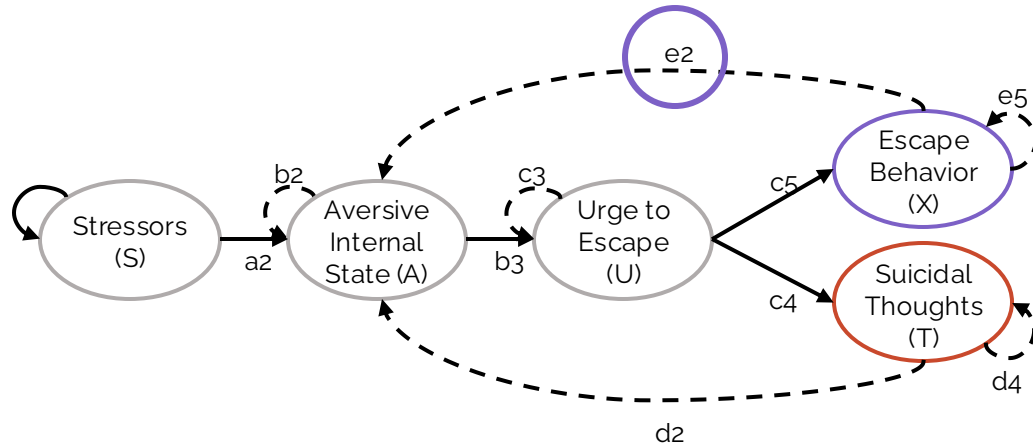
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# Simulating a case

Escape behaviors ARE NOT effective: suicidal thoughts emerge

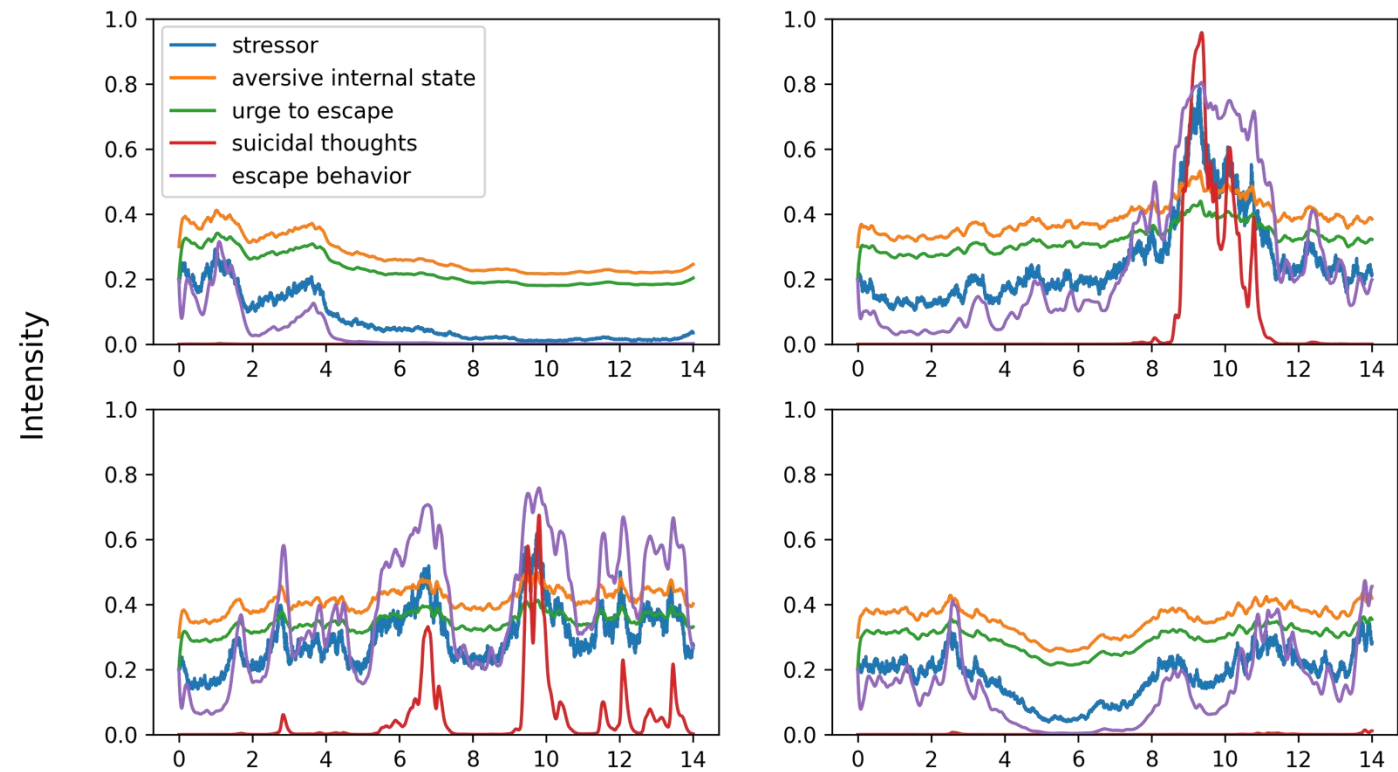
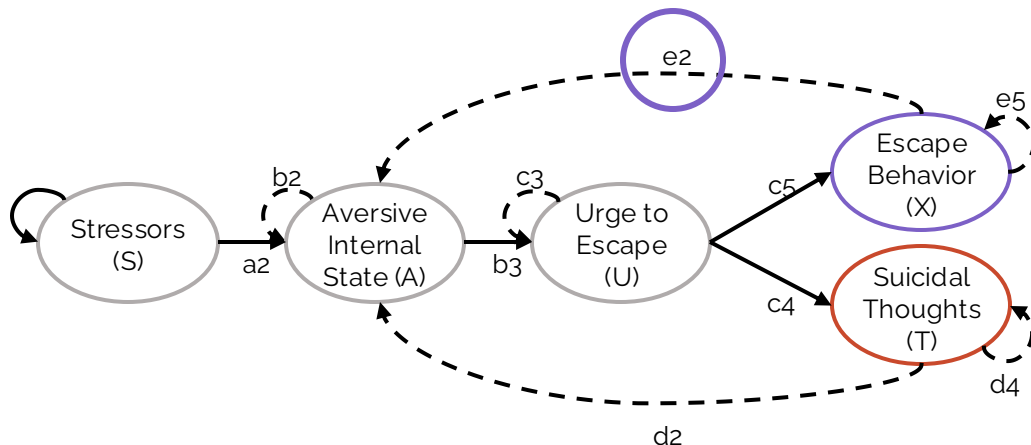
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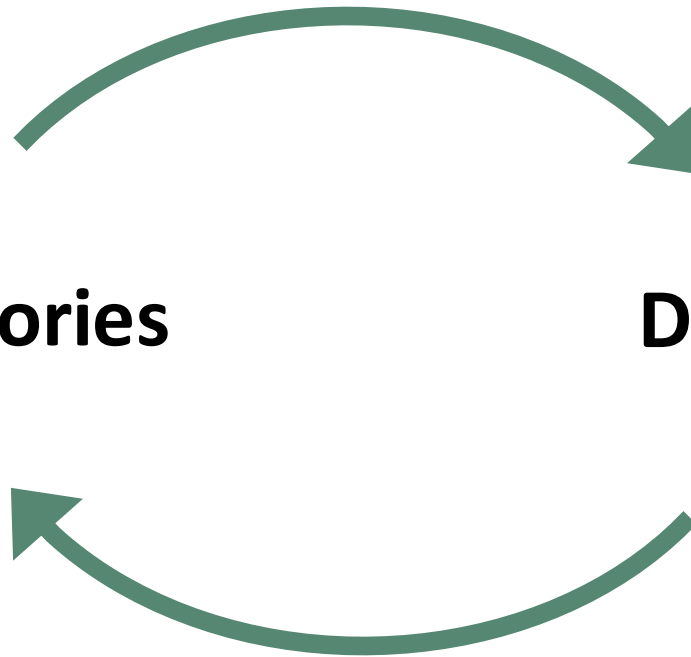
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**Theories**

**Data**



### Formal Theory

$$S_t = S_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t}$$

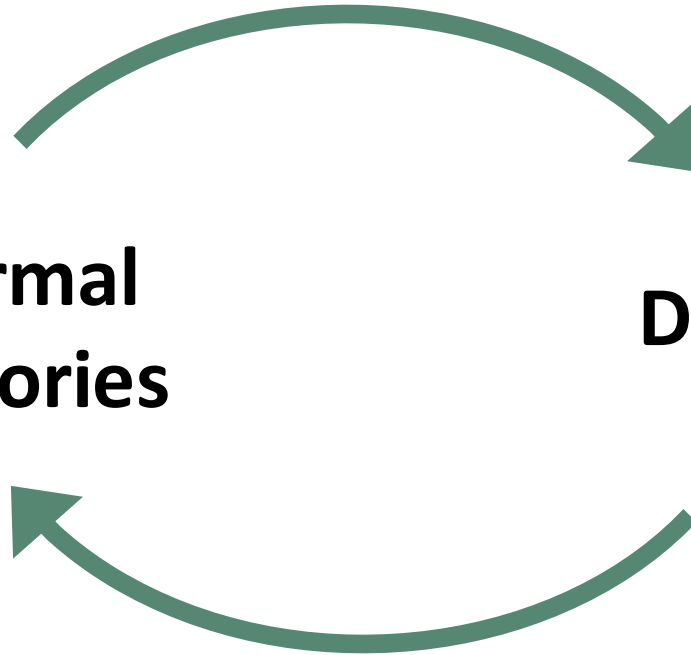
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**Formal  
Theories**

**Data**



# Epistemic iteration

## Formal Theory

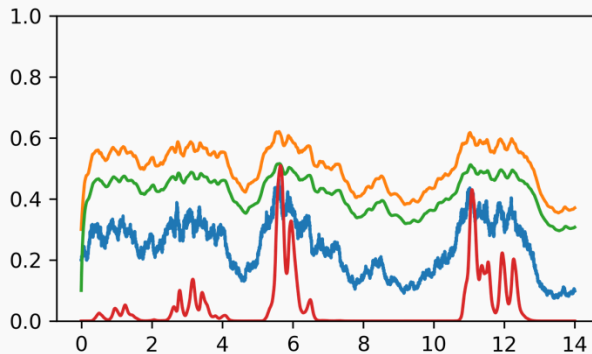
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## Simulations



Formal  
Theories

Data

# Epistemic iteration



Shari Hamm

## Formal Theory

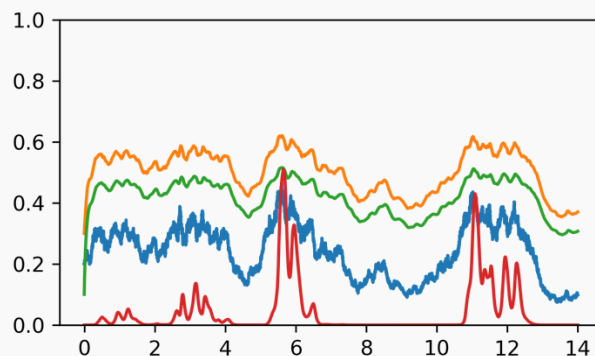
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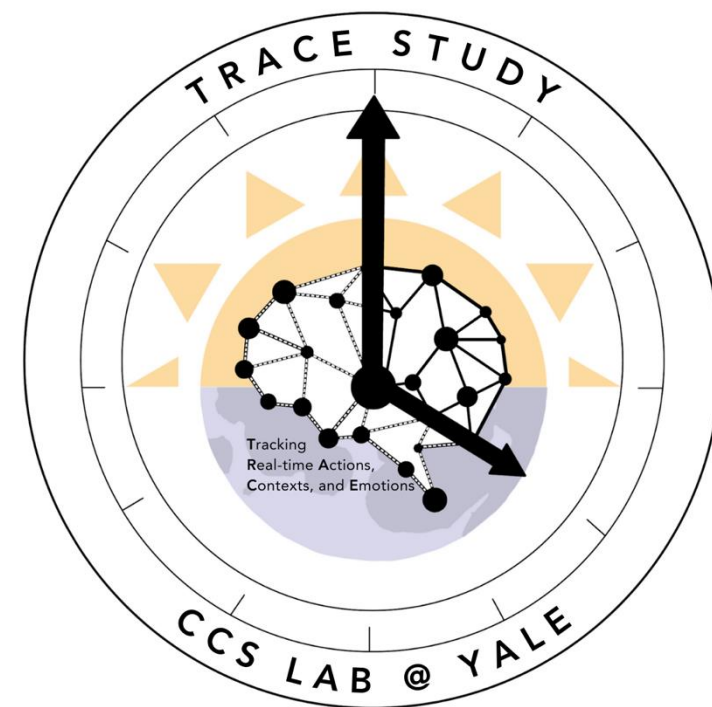
$$\frac{dT}{dt} = -d_4 T + \frac{1}{1 + e^{-c_{41}(U - c_{42})}}$$

## Simulations



Formal  
Theories

Digital  
Phenotyping



# Thank you!



Computational  
Clinical Science Lab

Capturing and modeling the  
complex dynamics of mental health



Shari  
Hamm



Leily  
Behbehani



Adanya  
Johnson



Grace  
Hart



## CCS Lab

Sharina Hamm

Leily Behbehani

Adanya Johnson

Grace Hart

Coby Barrow

Gaeun Gwon Lee

Brandon Felcher

Hannah Owens Pierre



## Collaborators

Kathryn Fox (Univ of Denver)

Annie Haynos (VCU)

Matthew Nock (Harvard)

Jordan Smoller (MGH)

Ruben Van Genugten (Northeastern)

Donald Robinaugh (Northeastern)

Rebecca Fortgang (MGH)

Alexander Millner (Harvard)

Christine Cha (Yale)



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